Int J Theor Phys (2010) 49: 1127–1132
DOI 10.1007/s10773-010-0293-6

Axially Symmetric Bianchi Type-I Model with Massless Scalar Field and Cosmic Strings in Barber's Self-Creation Cosmology

K.S. Adhav · A.S. Nimkar · V.G. Mete · M.V. Dawande

Received: 4 December 2009 / Accepted: 23 February 2010 / Published online: 4 March 2010

Abstract Axially symmetric Bianchi type-I space time is considered in the presence of massless scalar field and cosmic strings in Barber's (Gen. Relativ. Gravit. 14:117, 1982) self-creation theory with two conditions (i) $A = B^n$ and (ii) $\varepsilon + \lambda = 0$. Some physical and kinematical properties of the model, thus obtained, are also discussed.

Keywords Massless scalar field · Cosmic strings · Barber's self-creation cosmology · Bianchi type-I space time

1 Introduction

Barber has invented two continuous self-creation theories. Barber's [1] first theory is a modification of Brans-Dicke [2] theory which is also modification of Einstein theory. While second theory is direct modification of general theory of relativity. Barber's first theory is not accepted as it severely violates the equivalence principle. However, in his second theory the gravitational coupling of Einstein's field equations is allowed to be a variable scalar on the space-time manifold. In this theory the scalar field does not gravitate directly but simply divides the matter tensor acting as a reciprocal gravitational constant. In the limit, coupling constant $\Lambda \to 0$, this theory tends to Einstein's theory in every respect.

The massless scalar field in relativistic mechanics yields some significant results regarding both the singularities involved and Mach's principle. Hawking and Ellis [3] proved that the flat Robertson Walker model with a massless scalar field can be a steady state model as $t \to \infty$. Pimentel [4], Soleng [5–7], Singh and Deo [8], Reddy [9, 10], Reddy et al. [11], Maharaj and Beeshan [12], Reddy and Venkateswarlu [13–15], Shanti and Rao [16] are some of the authors, who have investigated various aspects of Barber's self-creation theories. Mohanty [17, 18], Panigrahi and Sahu [19] have examined an anisotropic homogeneous Bianchi

K.S. Adhav (✉) · A.S. Nimkar · M.V. Dawande
Department of Mathematics, S.G.B. Amravati University, Amravati 444 602, India
e-mail: ati_ksadhav@yahoo.co.in

V.G. Mete
Department of Mathematics, R.D.I.K. College, Badnera (Rly), India

type-I cosmological micro and macro models in Barber's second theory of gravitation in the presence of massless scalar field interacted with perfect fluid.

The concept of string theory was developed to describe events at the early stages of the evolution of the universe. Kibble [20] and Vilenkin [21] believed that strings can be considered as one of the sources of density perturbations that are required for the formation of large scale structures in the universe. The study of string cosmological models was initiated by Vilenkin [22], Letelier [23], Krori et al. [24]. Bhattacharjee and Baruah [25], Reddy [26], Adhav et al. [27] studied the problem of comic strings in Bianchi type cosmologies with a self-interacting scalar field.

2 Field Equations and Model

We consider axially symmetric Bianchi type-I metric

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2), \tag{2.1}$$

where A and B are functions of cosmic time 't'.

The Einstein-Barber field equations in second self-creation theory are

$$G_{ij} \equiv R_{ij} - \frac{1}{2} R g_{ij} = -8\pi \phi^{-1}(T_{ij} + T_{ij}^s) \tag{2.2}$$

and

$$\Box\phi = \frac{8\pi\Lambda}{3} \quad \text{(trace of energy momentum tensor } T_{ij} + \text{trace of energy momentum tensor } T_{ij}^s). \tag{2.3}$$

T_{ij} is the energy momentum tensor for massless scalar field, T_{ij}^s is the energy momentum tensor for cosmic strings. $\Box\phi$ is the invariant D'Alembertian, 'ϕ' is the Barber's scalar, Λ is the coupling constant to be determined from experiment where $0 < |\Lambda| < (10)^{-1}$. In the limit $\Lambda \to 0$, this theory approaches to the Einstein's theory in every respect. Due to the nature of the space time Barber's scalar ϕ is a function of 't'.

In order to study the cosmological effects, the energy momentum tensor T_{ij} (Singh and Deo [8]) for a massless scalar field distribution is given by

$$T_{ij} = v_i v_j - \frac{1}{2} g_{ij} v_k v^k, \tag{2.4}$$

together with Klein-Gordon wave equations

$$g^{ij} v_{;ij} = \sigma. \tag{2.5}$$

Here the massless scalar field v and the source density σ are both functions of cosmic time t only. The semicolon (;) denotes the covariant differentiation.

The equation of motion $T^{ij}_{;j} = 0$ are consequences of the field equation (2.1).

The energy-momentum tensor for cloud of strings (Letelier [23]) is given by

$$T_{ij}^s = \varepsilon u_i u_j - \lambda x_i x_j \tag{2.6}$$

with $\varepsilon = \varepsilon_p + \lambda$, where ε is the energy density, λ is the tension density of the string cloud and ε_p is the density of particles.

The string source is taken along x-axis which is the axis of symmetry.

Thus we have

$$u_i u^i = -1 = -x_i x^i \quad \text{and} \quad u_i x^i = 0, \tag{2.7}$$

where u_i is the four velocity for the cloud and x^i is the direction of the strings.

Using (2.4) to (2.7), the set of field equations (2.2) and (2.3) for the space time equation (2.1) reduce to following:

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} = -8\pi\phi^{-1}\left[\frac{1}{2}\nu_4^2 - \lambda\right], \tag{2.8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -4\pi\phi^{-1}\nu_4^2, \tag{2.9}$$

$$2\frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} = -8\pi\phi^{-1}\left[-\frac{1}{2}\nu_4^2 - \varepsilon\right], \tag{2.10}$$

$$\phi_{44} + \left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\phi_4 = -\frac{8\pi}{3}\Lambda[\nu_4^2 - \lambda - \varepsilon]. \tag{2.11}$$

The subscript 4 denotes ordinary differentiation with respect to t.

Again equations (2.8) to (2.11) are four highly non-linear differential equations in seven unknowns $A, B, \phi, \Lambda, \varepsilon, \lambda, \nu$. For complete determinacy of the system, two extra conditions are needed. For this purpose, first we assume the relation between metric potentials A and B as

$$A = B^n, \quad \text{where } n \text{ is real number}. \tag{2.12}$$

And secondly, we assume that the sum of the rest energy density ε and the tension density λ for a cloud of strings vanishes. The strings are Reddy strings i.e.

$$\varepsilon + \lambda = 0 \quad \text{(Reddy strings [26])}. \tag{2.13}$$

Equations (2.8) and (2.10) together with the conditions (2.12) and (2.13) lead to

$$\frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} = 0. \tag{2.14}$$

Using (2.12), it reduces to

$$\frac{B_{44}}{B} + (n+1)\frac{B_4^2}{B^2} = 0 \tag{2.15}$$

which further leads to

$$\log B_4 + (n+1)\log B = \log\alpha, \tag{2.16}$$

where $\log\alpha$ is the constant of integration.

Solving (2.16), we obtain,

$$A = (n+2)^{\left(\frac{n}{n+2}\right)}(\alpha t + \beta)^{\left(\frac{n}{n+2}\right)}, \qquad B = (n+2)^{\left(\frac{1}{n+2}\right)}(\alpha t + \beta)^{\left(\frac{1}{n+2}\right)}, \quad n \neq -2. \tag{2.17}$$

Using (2.13) and (2.17), equation (2.11) reduces to

$$\phi_{44} + \left(\frac{\alpha}{\alpha t + \beta}\right)\phi_4 = -\frac{8\pi\Lambda}{3}\nu_4^2. \tag{2.18}$$

Using (2.17), equation (2.9) becomes

$$\nu_4^2 = \frac{1}{4\pi}\left[\frac{(2n+1)}{(n+2)^2}\frac{\alpha^2}{(\alpha t + \beta)^2}\right]\phi. \tag{2.19}$$

Using (2.17) and (2.19) with substitution $\alpha t + \beta = T$, equation (2.18) reduces to the form

$$T^2\phi_{TT} + T\phi_T + K^2\phi = 0 \tag{2.20}$$

where

$$K^2 = \frac{2\Lambda}{3}\left[\frac{(2n+1)}{(n+2)^2}\right], \quad 0 < \Lambda < 10^{-1},\ n \neq -2.$$

On integration, (2.20) yields two basic solutions for ϕ as

$$\phi_1 = \cos(K\log T), \tag{2.21}$$

$$\phi_2 = \sin(K\log T). \tag{2.22}$$

The value ϕ_2 [second value of ϕ given in (2.22)] is not of importance as it leads to the unphysical situation.

From (2.19), we get

$$\nu_T = \gamma_1\frac{\sqrt{\phi}}{T} \tag{2.23}$$

where $\gamma_1 = \sqrt{\frac{\alpha^2}{4\pi}\frac{(2n+1)}{(n+2)^2}}$

$$\therefore \nu = \gamma_1\int\frac{\sqrt{\phi}}{T}dT + \gamma_2, \tag{2.24}$$

where γ_2 is constant of integration.

Using (2.21) and (2.22), we obtain the expressions for massless scalar field ν

$$\left.\begin{array}{l} \nu = \gamma_1\displaystyle\int\frac{\sqrt{\cos(K\log T)}}{T}dT + \gamma_2 \\ \text{and} \\ \nu = \gamma_1\displaystyle\int\frac{\sqrt{\sin(K\log T)}}{T}dT + \gamma_2 \end{array}\right\}. \tag{2.25}$$

From (2.5), the source density of the massless scalar field ν

$$\sigma = -\alpha^2\left(\nu_{TT} + \frac{\nu_T}{T}\right). \tag{2.26}$$

Using (2.25) and (2.26), we get

$$\sigma = \frac{\alpha^2\gamma_1 K}{2T^2}\frac{\sin(K\log T)}{\sqrt{\cos(K\log T)}}. \tag{2.27}$$

The energy density ρ associated with the massless scalar field ν (Mohanty [17, 18]; Panigrahi and Sahu [19]) is given by

$$\rho = \frac{\upsilon_4^2}{2} = \frac{\upsilon_T^2}{2} = \frac{1}{2}\frac{\gamma_1^2}{T^2}\cos(K\log T). \tag{2.28}$$

Substituting the values from (2.17), (2.19) and (2.21) in (2.8), we get, string tension density $\lambda = 0$.

Which further yields [using (2.13)] that rest energy density (for strings) $\varepsilon = 0$. Hence particle density (for strings) $\varepsilon_p = 0$.

It is interesting to note that cosmic strings does not exists in axially symmetric Bianchi type-I model with massless scalar field in Barber's self creation cosmology.

The axially symmetric Bianchi type-I model having massless scalar field coupled with cosmic strings in Barber's self-creation theory will be

$$ds^2 = -\frac{dT^2}{\alpha^2} + (n+2)^{(\frac{2n}{n+2})}(T)^{(\frac{2n}{n+2})}dX^2 + (n+2)^{(\frac{2}{n+2})}(T)^{(\frac{2}{n+2})}[dY^2 + dZ^2], \quad n \neq -2. \tag{2.29}$$

Above model is similar to the model obtained by Adhav et al. [27].

3 The Physical and Kinematical Properties

$$\text{Spatial volume} \quad = \frac{(n+2)}{\alpha}T, \tag{3.1}$$

$$\text{Expansion scalar} \quad \theta = \frac{1}{3T}, \tag{3.2}$$

$$\text{Shear scalar} \quad \sigma^2 = \frac{7}{162T^2}, \tag{3.3}$$

$$\text{and} \quad \lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0. \tag{3.4}$$

The deceleration parameter q is given by Feinstein et al. [28]

$$q = -[9\alpha + 1]. \tag{3.5}$$

The Barber's scalar ϕ, the scalar field ν, the source density σ and rest energy density ρ are given by (2.21) and (2.22), (2.25), (2.27), (2.28) respectively.

(a) When $T = 1$, we get $\phi = 1$ or 0, $\nu =$ constant, $\rho = 0$ or constant and $\sigma = 0$ or $\pm\infty$. In this case the space time reduces to a flat space time.
(b) When $T \to 0$ or ∞, then the quantities ν, ϕ, ρ and σ are undetermined. The metric potentials A and B tend to zero as $T \to 0$. Therefore, the spacetime collapses at $T = 0$ and admits a singularity at $T = \infty$.
(c) When the coupling parameter $\Lambda \to 0$ ($T \neq 0$ or ∞) then $\phi \to 1$, $\nu \to$ constant, $\sigma \to 0$ and $\rho \to$ constant. i.e. In the limit $\Lambda \to 0$, Barber's theory approaches to Einstein theory of relativity.
(d) The spatial volume tends to zero as $T \to 0$ and spatial volume tends to $\pm\infty$ as $T \to \pm\infty$. These results show that the universe starts expanding with zero volume and blows up at infinite past and future.

(e) One can observe that expansion scalar $\theta \to 0$ as $T \to \infty$ and $\theta \to \infty$ as $T \to 0$. Thus the universe is expanding with increase of time but the rate of expansion becomes slow as time increases (where $k_1 + 2k_2 = 1$).
(f) Shear scalar $\sigma^2 \to 0$ as $T \to \infty$ and $\sigma^2 \to \infty$ as $T \to 0$. Thus the shape of the universe changes uniformly. It is observed that $\lim_{T\to\infty}(\frac{\sigma}{\theta}) \neq 0$ which confirms that the universe remains anisotropic throughout the evolution.

4 Conclusion

In this paper, we have obtained the anisotropic homogeneous axially symmetric Bianchi-I cosmological model in the presence of massless scalar field and cosmic strings with conditions $A = B^n$ and $\varepsilon + \lambda = 0$ (Reddy strings [26]) in Barber's self-creation theory. Model is expanding, shearing, non rotating and do not approach isotropy for large value of time T.

It is interesting to note here that Barber scalar $\phi \to 1$, massless scalar field $\nu \to$ constant, source density $\sigma \to 0$ and energy density of scalar field $\rho \to$ constant as the coupling parameter $\Lambda \to 0$ ($T \neq 0$ or ∞). Therefore, in the limit $\Lambda \to 0$, Barber's theory approaches to Einstein theory of relativity.

References

1. Barber, G.A.: Gen. Relativ. Gravit. **14**, 117 (1982)
2. Brans, C., Dicke, R.H.: Phys. Rev. **124**, 925 (1961)
3. Hawking, S.W., Ellis, G.F.R.: The Large Scale Structure of Space Time, p. 94. Cambridge University Press, Cambridge (1973)
4. Pimentel, L.O.: Astrophys. Space Sci. **116**, 395 (1985)
5. Soleng, H.H.: Astrophys. Space Sci. **139**, 373 (1987)
6. Soleng, H.H.: Astrophys. Space Sci. **102**, 67 (1987)
7. Soleng, H.H.: Acta Phys. Hung. **48** (1988)
8. Singh, R.K.T., Deo, S.: J. Math. Phys. **27**, 1074 (1986)
9. Reddy, D.R.K.: Astrophys. Space Sci. **132**, 401 (1987)
10. Reddy, D.R.K.: Astrophys. Space Sci. **133**, 189 (1987)
11. Reddy, D.R.K., Avadhanulu, M.B., Venkateswarlu, R.: Astrophys. Space Sci. **134**, 201 (1987)
12. Maharaj, S.D., Beesham, A.: Astrophys. Space Sci. **140**, 33 (1988)
13. Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **151**, 353 (1989)
14. Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **152**, 337 (1989)
15. Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **168**, 135 (1990)
16. Shanti, K., Rao, V.U.M.: Astrophys. Space Sci. **179**, 147 (1991)
17. Mohanty, G.: Bull. Inst. Math. Acad. Sin. (ROC) **28**, 43 (2000)
18. Mohanty, G.: Astrophys. Space Sci. **281**, 633 (2002)
19. Panigrahi, U.K., Sahu, R.C.: Theor. Ref. Appl. Mech. **30**(3), 163 (2003)
20. Kibble, T.W.B.: J. Phys. A **9**, 1387 (1976)
21. Vilenkin, A.: Phys. Rev. **121**, 263 (1985)
22. Vilenkin, A.: Phys. Rev. D **23**, 852 (1981)
23. Letelier, P.S.: Phys. Rev. D **28**, 2414 (1983)
24. Krori, K.D., Chaudhari, T., Mahanta, C.R.: Gen. Relativ. Gravit. **22**, 123 (1990)
25. Bhattacharjee, R., Baruah, K.K.: Gen. Relativ. Gravit. **26**, 647 (2001)
26. Reddy, D.R.K.: Astrophys. Space Sci. **300**, 381 (2005)
27. Adhav, K.S., Nimkar, A.S., Dawande, M.S.: Int. J. Theor. Phys. **47**, 3201 (2008)
28. Feinstein, A., Ibanez, J., Labarge, P.: J. Math. Phys. **36**, 4962 (1995)